Patent Thickets:
Strategic Patenting of Complex Technologies

By James Bessen*

Abstract: Patent race models assume that an innovator wins the only patent covering a product. But when technologies are complex, this property right is defective: ownership of a product's technology is shared, not exclusive. In that case I show that if patent standards are low, firms build "thickets" of patents, especially incumbent firms in mature industries. When they assert these patents, innovators are forced to share rents under cross-licenses, making R&D incentives sub-optimal. On the other hand, when lead time advantages are significant and patent standards are high, firms pursue strategies of "mutual non-aggression." Then R&D incentives are stronger, even optimal.

Keywords: patents, technological innovation, intellectual property
JEL codes: K3, L1, O3

Affiliations: Research on Innovation and Boston University School of Law (Visiting Researcher)

jbessen@researchoninnovation.org

Introduction

At first glance, some firms seem to behave irrationally with regard to patents. Consider Oracle Corporation, the software firm that has developed leading, innovative database management systems since 1979. Oracle could have obtained a large portfolio of patents on many of its well-known innovations, but chose not to. Although patents have been issued on database techniques at least since the early 1970s, and especially since 1990, Oracle did not obtain its first patent until 1995 and acquired only 161 patents during the 1990s. This was a conscious strategy as described by Jerry Baker, Senior Vice President of Oracle:

> Our engineers and patent counsel have advised me that it may be virtually impossible to develop a complicated software product today without infringing numerous broad existing patents. …As a defensive strategy, Oracle has expended substantial money and effort to protect itself by *selectively* applying for patents which will present the best opportunities for cross-licensing between Oracle and other companies who may allege patent infringement. If such a claimant is also a software developer and marketer, we would hope to be able to use our pending patent applications to cross-license and leave our business unchanged. (USPTO Hearings, 1994, my emphasis)

Other software firms have also obtained a minimal number of "defensive" patents—on average, software firms acquire only a quarter as many patents relative to R&D as other firms (Bessen and Hunt, 2003). Like Oracle, they neither aim to assert patents against (non-suing) competitors, nor do they license all patents that they might possibly infringe, in the hope that other firms will follow a similar strategy. Such a strategy of "mutual non-aggression" or "mutual forbearance" was also common in the early semiconductor and computer industries.[1]

The problem Baker describes is often called a "patent thicket." These occur when each product may involve many patents, in contrast with the one-to-one correspondence between products and patents that is assumed in the patent race literature. Recent commentators suggest that lower patenting standards encourage patent thickets, creating difficulties for innovators (see Gallini, 2002, for a review). When innovators must negotiate with large numbers of patentholders, they may face excessive transaction costs (Heller and Eisenberg, 1998), "holdup," and problems of vertical monopoly (Shapiro, 2001). Some researchers propose that cross-licensing and patent pools may resolve these problems (Merges, 1999, Shapiro, 2001, Lerner and

---

[1] In the semiconductor industry prior to the 1980s, firms licensed key technologies from one or two large firms (AT&T initially, then Fairchild and Texas Instruments), but often did not engage in much additional active licensing (Tilton, 1971, Levin, 1982). Levin writes (1982) "…many semiconductor firms simply neglect to arrange to license all the patents that might conceivably be used to block its activity. Typically, most merchant firms regularly review cross-licensing arrangements with technologically progressive captive suppliers like IBM and AT&T [who produce only for their own use], but they often infringe on one another's patents. A patent holder typically ignores such infringement, since it is likely to be guilty of reciprocal offense." This behavior has since been replaced by more aggressive cross-licensing (Grindley and Teece, 1997, Hall and Ziedonis, 2001).

Tirole, 2002). This view differs sharply from earlier skepticism that these institutions might be anti-competitive.

But there is still reason to be skeptical about cross-licensing. This paper argues that patent thickets can reduce R&D incentives even when there are no transaction costs, holdup or vertical monopoly problems. There are *different types* of cross-licensing strategies with sharply different normative implications. Indeed, in contrast to Oracle, which views cross-licensing as an unfortunate last resort, other companies aggressively seek to build large patent portfolios and to use them to extract benefits from competitors. For instance, IBM acquired nearly one hundred times as many patents as Oracle during the 1990s although IBM's R&D budget is only about five times as large as Oracle's. IBM expresses their strategy this way:

> The IBM patent portfolio gains us the freedom to do what we need to do through cross-licensing—it gives us access to the inventions of others that are key to rapid innovation. Access is far more valuable to IBM than the fees it receives from its 9,000 active patents. There's no direct calculation of this value, but it's many times larger than the fee income, perhaps an order of magnitude larger. (Smith, 1990)

Note that IBM now earns nearly two billion dollars per year in royalties. Survey evidence finds that many other firms also obtain patents in order to "block competitors" (Cohen et al, 2000).

Also, note that these are expressly patent *portfolio* strategies—firms interact over entire portfolios rather than over individual patents. Instead of licensing carefully chosen individual patents—as in most of the economics literature on licensing—firms in semiconductors, electronics and computers negotiate based on the relative heights of their stacks of all related patents, and they license entire portfolios for a technology field, including patents for which they have not yet filed applications (Grindley and Teece, 1997, Hall and Ziedonis, 2001).

This paper builds a simple two-firm innovation race model where both mutual non-aggression strategies and aggressive portfolio cross-licensing strategies emerge as equilibrium strategies. I intend this model to explain behaviors observed in industries with complex technologies.[2] It differs from standard patent race models in two principal ways. First, I model patenting as separate from, but complementary to, the performance of R&D necessary to develop a product. Because there are many possible patents associated with each product, firms can put more or fewer resources into obtaining patents for any given level of product-related R&D. Second, I assume that patent infringement and validity are uncertain. A single patent does not

---

[2] In industries with so-called "discrete" technologies, such as chemicals, firms may have largely non-overlapping patent portfolios. They do not, in general, practice cross-licensing of whole portfolios, but some forms of portfolio based patenting do occur, including "evergreening" and patent "fencing."

convey certain monopoly rights, rather each patent provides a positive, but less than certain, probability of winning an infringement suit. This means that the more patents a firm has related to a given product, the greater the joint probability of prevailing at trial. Consequently, a firm with a larger patent portfolio has a greater probability of winning, all else equal, and hence a stronger bargaining position in cross-licensing negotiations.

In this model, patenting itself is an economic activity, trading off the probability of winning at litigation against the cost of patenting. Moreover, patentability standards affect the cost of obtaining a patent and, thus, they determine the cost of building a patent portfolio. When patent standards are high, portfolio building is costly, and firms are more likely to follow a strategy where they do not assert their patents. On the other hand, low patenting standards are associated with a larger number of patents and aggressive assertion of patents, leading to cross-licensing.

When firms do not assert their patents, innovators may still realize rents because of lead time advantages. Empirical research finds that firms see lead time advantages and related advantages as strong sources of appropriability—stronger than patents, in fact, except in the pharmaceutical and chemical industries (Levin et al, 1987, Cohen et al, 2000). Lead time advantages provide incentives for R&D, and, if the first innovators garner a large enough share of potential rents, then R&D levels may be optimal or even supra-optimal.

On the other hand, when firms cross-license, I find that R&D incentives are sub-optimal under general conditions. In contrast to simple "winner-take-all" patent race models, the rewards to innovating are shared with other patent holders. In particular, cross-licensing sharply reduces the incentive effect of lead time advantages because the winner's profits are included in the bargaining over a cross-license and are shared in the bargaining solution. In addition, aggressive cross-licensing involves a higher level of socially wasteful patenting activity. I find that social welfare is higher when firms choose mutual non-aggression, except when lead time advantages are small or non-existent. Thus I conclude that, far from remedying the problems caused by low patenting standards, aggressive portfolio cross-licensing may be evidence that low patent standards are inducing socially wasteful behavior and reducing R&D incentives.

Finally, I find that aggressive cross-licensing affects firms and industries unevenly. Cross-licensing is more likely in mature industries with established products. Moreover, incumbent firms obtain a greater share of profits under cross-licensing and, under general conditions, their profits are greater when patenting standards are lower.

This paper addresses a different aspect of patent thickets than some of the previous literature. Heller and Eisenberg (1998) raise the concern that transaction costs may become prohibitive when firms must bargain with many different patent holders to obtain the rights needed to make a product. Merges (1999) argues that patent pools may overcome this contracting problem. Patent pools have been formally modeled by Shapiro (2001) and Lerner and Tirole (2002) as a problem of vertical monopoly. They specify conditions where multiple firms infringe a discrete set of patents with near certainty.

In contrast, my model concerns horizontal technological competition and bilateral negotiation between two firms with uncertain conflicting patents. I identify a problem that occurs even without transaction costs, with as few as two firms and without holdup.[3]

My model also shares a perspective with Kitch's "prospect" theory of patents (1977). In the prospect model, as well as in this paper, a patent does not embody all the knowledge necessary to produce a successful commercial product. However, Kitch implicitly assumes that a *single* patent covers the entire field of knowledge necessary to develop a commercial product; this allows the patent holder to exercise exclusive control over commercialization. Kitch argues that exclusive control provides the most efficient incentives for commercialization.[4] In my model, *many* patents and many patent holders may be involved in developing a commercial product. Because no single patent holder exercises exclusive control, Kitch's argument does not hold in this case.

The paper is organized as follows. The next section presents the model. Section II presents results for the basic innovation race model. Section III extends the model and section IV concludes.

I. Model

I consider a model with two risk neutral firms, A and B.[5] In a simple multi-stage patent race model, in the first stage, both firms might sink R&D, x_A and x_B, then "race" in continuous time to discover a single invention, as in Loury (1979). In the final stage, the winner of the innovation race obtains a patent and earns monopoly profits, V^M.

---

[3] It also differs from models of the cross-licensing of individual patents (Eswaran, 1992, Fershtman and Kamien, 1993).

[4] This argument assumes no transaction costs and no asymmetric information regarding the licensing of the patent. This may not be realistic. See Bessen (2003).

[5] I also modeled a multi-firm model with free entry. Many of the basic intuitions developed for the two firm case still hold, however, results are sensitive to the particular specification of the entry constraint. For simplicity, I present only the two firm case here.

The game here is a bit more complicated. Patenting strategies add an additional dimension to each firm's initial decisions. Firms may make different R&D investments depending on whether they intend to assert their patents after innovation or not. This decision will, in turn, depend on the expected returns to cross-licensing or litigation compared to the returns when firms do not assert their patents.

The stages of the game are as follows (see Figure 1):

1. In the first stage, firms sink R&D investments, x_A and x_B and they begin acquiring patents. Each firm can observe the rate at which the other firm acquires patents (I explain the significance of this below).

2. The firms acquire n_A and n_B patents, respectively, each at total cost C_i, $i = A, B$. These costs are separate from the costs of performing the product R&D, x_i, although they may also involve engineering resources and are related (see below). This sunk cost can credibly commit a firm to an assertive strategy. In this stage, the firms then conduct a Poisson innovation race where the Poisson hazard depends on the R&D investments, x_i. The stage ends when one firm makes a successful innovation.

3. The firms then decide whether they wish to actually assert their patents or not. If neither firm chooses to assert its patents, then the innovating firm goes into production, followed by the other firm, which imitates sometime later (allowing the innovator to capture a lead time advantage). If one or both firms assert patents, then the firms negotiate a cross-license. If negotiations fail, they go to court. The probability of winning at litigation depends on each firm's portfolio size, n_i (see below).

This stage structure reflects that, with complex technologies, firms obtain patents to protect a portion of the technology before they develop *all* of the knowledge necessary for commercial success. Firms logically behave differently during the R&D stage if they intend to assert their patents or if they expect their competitor to assert its patents.

Some details of the model are as follows:

In practice, firms take considerable time to acquire a large portfolio of patents. I assume that firms cannot vary the rate of patent acquisition substantially without incurring large adjustment costs. Because players observe their opponent's rate of patent acquisition in Stage 1, they can infer whether their opponent is committing to an aggressive strategy. If so, a player has sufficient time in Stage 2 to accumulate a substantial portfolio. This feature of the model avoids

unrealistic timing games where a player might seek to surprise an opponent by accumulating a large portfolio overnight.

Let the ith firm's total cost of a patent portfolio depend on the number of patents, n, R&D, x, and also on "patenting standards," S, that is, $C_i = C(S, n_i, x_i)$. Patenting and product R&D are complementary activities in the following sense: greater R&D expenditure may reduce the cost of obtaining a patent. This makes sense because product R&D may generate patentable ideas or otherwise reduce the engineering resources necessary to obtain a patent.[6] The variable S includes all policy features that affect the cost of patenting.[7] I assume that

(1)
$$\frac{\partial C}{\partial S}, \frac{\partial C}{\partial n_i} > 0 \geq \frac{\partial C}{\partial x_i}, \qquad \frac{\partial^2 C}{\partial x_i^2}, \frac{\partial^2 C}{\partial n_i^2} \geq 0, \qquad \frac{\partial^2 C}{\partial S \partial x} = 0,$$
$$C(0, x, n) = C(S, x, 0) = 0$$

The second stage concludes with the innovation race. I assume Poisson hazards $h(x_A)$ and $h(x_B)$, which are the probabilities that each firm will make a successful innovation between times t and $t + dt$. I assume h is increasing, concave and $h(0) = 0$. Time is discounted at rate r. This discounting means that the expected speed of innovation, determined by investment levels, affects social welfare.

In the third stage, one or both firms may decide to assert their patents against the other firm. If this occurs,[8] then the firms either litigate or cross-license. The value of a patent consists entirely of its potential ability to block a competitor through litigation (including counter-suits over validity and infringement).[9] I assume two conditions that give rise to the strategic licensing of whole portfolios. First, the outcome of litigation is uncertain, with probability q that a firm may be able to win its suit against its competitor (but not necessarily the counter-suit) such that

---

[6] Patents may be taken out on inventions that the firm actually uses in its product and also on alternative techniques. The firm may choose to patent alternative techniques in order to strategically block competitors. So, while R&D may generate some patents that require little additional engineering, a large portfolio with many alternative techniques may require a large investment in engineering beyond that required for the product itself. Note that the cost of patenting with zero R&D may be prohibitively high.

[7] This includes patent application and renewal fees as well as standards of "non-obviousness," novelty, utility, subject matter, etc. that may impose additional R&D costs on a firm seeking to obtain a patent portfolio of a given size. Patent term affects cost in a dynamic model where firms seek to maintain an optimal portfolio size. Patent scope and patent enforcement (litigation costs) affect the appropriability each patent conveys, thus affecting the effective cost of quality-adjusted patents. I set the cross derivative of cost with respect to S and x to zero for simplicity; this is likely to be quite small.

[8] It is possible that a firm could decide to assert its patents in stage one, yet not want to assert them in stage three. In equilibrium this does not occur, so to simplify the exposition I do not discuss this possibility.

[9] In a slightly more complicated version of the model, firms may also obtain patents to block imitators who do no independent R&D. In that version, given a non-negative level of imitation costs, no imitators will choose to enter as long as each firm that invests in R&D obtains a minimum number of patents. Since I am concerned mainly with modeling the interaction between firms that do R&D, I effectively assume this minimum number to be zero without loss of significant generality.

$q < 1$. Second, firms can potentially obtain patents on a large number of inventions related to a single product and these increase the probability of litigation success, although with diminishing returns. That is, q, is an increasing concave function of the firm's patent portfolio size, n, $q = q(n)$. I also assume, without loss of generality, that $q(0) = 0$. The winner(s) of litigation are able to block competitors and claim any profits earned before and after litigation; I ignore other penalties and implicit costs such as those imposed by preliminary injunctions.

There are then four random outcomes to litigation, determined by the independent probabilities, $q(n_A)$ and $q(n_B)$. Temporarily ignoring lead time advantage, the associated profit outcomes are: if both firms lose, they each earn V^C, the competitive level of profits; if just one firm wins, it earns monopoly profits V^M and the other earns 0; and if both firms win, they bargain. In the last case, I assume that firms bargain with symmetric threat points and symmetric bargaining power, each realizing licensing profits of V^L. Below, in Section III, I consider the case where one firm is an incumbent and thus has a different threat point. In the current case, these outcomes are:

	B loses	B wins
A loses	$V^C,\ V^C$	$0,\ V^M$
A wins	$V^M,\ 0$	$V^L,\ V^L$

Without loss of significant generality, I assume that licensing is fully efficient (no dissipation of rents) so that $V^L = \frac{1}{2}V^M$. Considering consumer surplus and the dissipation of rents,

(2) $0 \le V^C \le V^L = \frac{1}{2}V^M \le \frac{1}{2}V^{Soc}$

where V^{Soc} is social surplus and this exceeds monopoly profits if there is net consumer surplus.

The expected outcome from litigation serves as a "threat point" to the bargaining problem for the cross-license in stage three. Under license, the joint profit of the two firms is $2V^L$. I assume that this is larger than the joint expected profit from litigation (say legal costs are sufficiently large), so that if firms assert patents, they will always negotiate a cross-license rather than litigate. Assuming equal bargaining power, the Nash bargaining solution splits the net bargaining surplus.

Finally, note that I have structured the game so that holdup does not occur. Holdup might arise if the innovating firm went into production *before* the firms negotiated a license. If production required fixed costs, then the producing firm would be at a disadvantage during the bargaining process. Knowing this, firms would choose to negotiate before going into production. In this model, the first innovator knows the other firm's intent to assert patents and so it can initiate bargaining. In a more complex model, firms may lack such information. As Jerry Baker of Oracle suggests, firms may acquire patents to protect against a probability that some unidentified firm may assert patents. Assuming full information about the identity of potential legal adversaries simplifies the model; it does mean, as we shall see, that under mutual non-aggression firms don't acquire any patents while in practice the mutual non-aggression strategy involves a limited number of defensive patents. Firms may also acquire a limited number of patents to protect against knock-off imitation (see footnote 9).

Lead time advantage

The exposition so far has ignored lead time advantage. Lead time advantage arises naturally in a model where imitation requires time or stochastic discovery. Consider first the situation where firms do not assert their patents. One firm innovates first followed by the second firm at a later time and the first firm earns monopoly rents during the interim. I assume that the associated rents are an exogenous feature of the industry.[10] Let f represent the portion of present value captured during this interim period. Then under duopoly competition, the winner of the innovation race receives $f V^M + (1-f) V^C$ and the loser receives $(1-f) V^C$. The first innovator gains additional profits relative to the second of $f V^M$. Call f the lead time advantage and $f V^M$ the value of the lead time advantage.

Now consider the case where firms assert patents. Since the negotiated license is determined by litigation outcomes, the value of the lead time advantage will be determined by the advantage that the first innovator realizes in the litigated outcome. Assume for the moment that litigation is concluded instantaneously before firms begin commercial production. If one firm wins at law, then the outcome of litigation is determined as above—the winner has a legal claim on any lead time profits earned as well as subsequent profits. If both firms win, then the loser of

---

[10] The lead time before imitation might be a function of industry spillovers, the difficulty of reverse engineering trade secrets or other industry characteristics. In a more complex model, the rents earned as a first innovator might also depend on the level of R&D investments and thus would be partially endogenous. This is a second order effect I ignore for simplicity of exposition.

the innovation race has a partial claim on lead time profits earned—these profits become part of the "pie" divided in the negotiated license agreement. So, in these cases, the first innovator does not realize a greater outcome. Only in the event that both firms lose at trial can lead time advantages come into play—this is just the duopoly case. The expected value of the lead time advantage in this case is $(1 - q(n_A))(1 - q(n_B)) f V^M$. Note that this value is typically much less than in the non-assertion case.

But the first innovator may not be able to realize even this much advantage if litigation takes time and the imitator can use this time to catch up. Once a suit is filed, plaintiffs may obtain preliminary injunctions.[11] More important, customers may be unwilling to purchase an innovative product that is threatened with an infringement judgment. Not only do customers risk losing the value of their purchase and related complementary investments, but they may also be liable for infringement themselves. Moreover, since the average patent suit takes thirty-one months (Magreb, 1993) and the loser of the innovation race has opportunities to delay the legal proceedings, the lead time advantage is unlikely to outlast a trial. The first innovator will be unable to realize significant advantage through litigation and the value of the lead time advantage under cross-license can be no greater.

As an alternative, firms might try to preserve the incentive effect of the lead time advantage by using a clever *ex ante* license.[12] Suppose, for example, the firms signed a contract in stage one, before sinking R&D investments, and this contract prohibited the loser of the innovation race from entering the market for *t* months after the innovation is made. The winner would be able to earn monopoly rents during this period and both firms would earn V^L thereafter. A difficulty with this sort of contract, however, is that it may be infeasible to describe *ex ante* what constitutes "winning" the innovation race (see Tirole 1999). Then firms can claim to have won with a mediocre improvement (hoping that they will be able to realize soon a truly successful innovation and earn monopoly rents for a portion of the *t* months), a third party cannot verify this claim, and so the true first innovator may get no particular incentive.

---

[11] If a plaintiff with a preliminary injunction loses at trial it must make up the defendant's lost profits, however, there is significant risk that the lead time advantage will not be fully compensated for in this case.

[12] This approach has the additional disadvantage that it may run afoul of antitrust regulations, but I will temporarily ignore that consideration and rule out such licensing on other grounds.

Given these various problems, the winner of the innovation race will at best be able to capture only a small fraction of the total lead time advantage in a cross-license, and this value will be less than in the non-assertion case:[13]

> Proposition 1. The value of the lead time advantage is greater when firms do not assert patents and is less when firms assert patents and outcomes are determined either under a cross-license or through litigation.

This result is important for the following reason. It is widely recognized that lead time advantages may make patents unnecessary in some cases. But since these advantages arise from the nature of market competition, they are often assumed to arise *independently* of patent policy. The result here, as elaborated below, implies that patent policy affects the value of the lead time advantage. Some patent policies may reduce this value and consequently reduce market-based incentives to innovate.

To simplify matters, I assume below that lead time advantages do not enter into the cross-licensing calculation. This means that R&D incentives are slightly understated for the case of cross-licensing, however, the general import of the model is unaffected.

II. Equilibrium Behavior and Normative Results

Equilibrium solutions

This game has a unique sub-game perfect Nash equilibrium in pure strategies that is one of three types:

1.   Deterrence, where one firm obtains so many patents that the other firm chooses not to enter. This equilibrium only exists for some range of parameters. I am interested in the range where deterrence does not occur, so I ignore this equilibrium.

2.   Mutual non-aggression, where neither firm patents.[14]

3.   Portfolio cross-licensing, where both firms build large portfolios and cross-license them to each other.

---

[13] One can envision other, more complex, models where, perhaps, a larger share of lead time advantage could be captured under a cross-license. For instance, if a cross-license term is five years, then a firm may capture lead time advantages on innovations occurring during the term of the license. But then firms may take such possibilities into account when constructing the license or, perhaps, when negotiating the subsequent license. Nevertheless, it does seem that much of the lead time advantage is lost simply because under a cross-license the other firm can lay claim to some share of it.

[14] As noted above, if firms needed to acquire a minimum number of patents to deter knock-off imitators, then they would acquire this (small) number instead of zero patents in this equilibrium.

The calculations of the equilibrium values are shown in the Appendix. For mutual non-aggression, the expected net discounted profit of each firm is $W^N = W^N(\hat{x}^N)$ where "N" signifies non-aggression and $\hat{x}^N$ is the firm's optimal level of R&D. This is determined by the first order condition:

$$(3) \qquad I^N \cdot h'(\hat{x}^N) = \left(r + 2h(\hat{x}^N)\right)^2, \qquad\qquad I^N \equiv (r + h(\hat{x}^N))f V^M + r (1 - f)V^C$$

The variable I^N may be thought of as the "R&D incentive." Given the concavity of h, the equilibrium R&D level is increasing in I^N. A larger incentive corresponds to more R&D.

For the cross-licensing equilibrium, $W^L = W^L(\hat{x}^L, \hat{n})$ where "L" signifies licensing, an additional first order condition determines the firm's optimal patent portfolio size, $\hat{n}$ (see Appendix), and the optimal level of R&D is determined by:

$$(4) \qquad I^L \cdot h'(\hat{x}^L) = \left(r + 2h(\hat{x}^L)\right)^2, \qquad\qquad I^L \equiv \frac{r V^L}{1 + \partial C/\partial x}$$

Given the structure of the game, in stage one both firms will select whichever strategy yields the greater equilibrium profits in stage three. For a given set of parameters, one can define a unique boundary value of the lead time advantage $f = \bar{f}(S, V^C/V^L)$

$$(5) \qquad \bar{f}(S, V^C/V^L) = f \text{ such that } W^N = W^L.$$

Then, if $f > \bar{f}$, firms will choose the non-aggression equilibrium and vice versa. This is shown in Figure 2, which plots a phase diagram. On the vertical axis, from zero to one, is the industry lead time advantage, f; on the horizontal access is the industry ratio of V^C/V^L. Then industries in the region above $\bar{f}(S_0, V^C/V^L)$ choose non-aggression while those below choose cross-licensing. In general terms, firms prefer cross-licensing when the lead time advantage is small and dissipation of rents from competition is large. Firms choose mutual non-aggression when there are large lead time advantages and/or little competitive dissipation of rents.

Patent Standards and Cross-Licensing

Numerous observers have suggested that patent thickets may be related to lower standards at the patent office (Gallini, 2002, Shapiro, 2001). Historically, the first evidence of strategic patent portfolio behavior and the first patents pools in the U.S. arose following a significant reduction in patent examination standards. The standards for patent examination were sharply reduced during the late 1840s and early 1850s when the scientists who conducted patent

examinations were replaced by "liberalizers" who accepted a much larger percentage of patent applications (Post, 1976). This change in standards was followed within a few years by evidence of patent thicket behavior. In 1856, a group of firms jointly acquired a dominant position in sewing machine patents and formed the first patent pool. Also during the 1850s, the Draper Company acquired a dominant patent position for loom temples and began a long term practice of aggressively using patent thickets (Draper, 1903, Mass, 1989).[15]

This suggests a possible connection between low patent standards and aggressive patent strategies. In fact, the following holds (see Appendix):

> Proposition 2. Cross-licensing and patent standards. As long as $V^C < \frac{1}{2} V^M$ and
> $$\frac{d\,C(S,\,\hat{n}(S),\,\hat{x}(S))}{d\,S} > 0,$$ an increase (decrease) in patenting standards, S, decreases
> (increases) the range over which cross-licensing occurs and increases (decreases) the range over which mutual non-aggression occurs.

The first condition is simply the existence of dissipation of rents under competition. The second condition means that the demand for patents is not highly elastic. Both are reasonable conditions. The intuition is then simple: cheap patents make patent thickets more affordable.

The effect of an increase in patentability standard S is shown in Figure 2, where $S_1 > S_0$. The region where cross-licensing occurs is reduced with a higher patenting standard.

Socially Optimal R&D

The optimal R&D investments allocated by a social planner can also be calculated. The first order condition is for each firm to invest $\hat{x}^{Soc}$ in R&D (see Appendix):

$$(6) \qquad I^{Soc} \cdot h'(\hat{x}^{Soc}) = \left(r + 2\,h(\hat{x}^{Soc})\right)^2, \qquad\qquad I^{Soc} \equiv r V^{Soc}$$

where "Soc" designates the social planner, and I^{Soc} is the R&D incentive

This can be compared with similar expressions in (3) and (4). Given this comparison,

> Proposition 3. Social efficiency of R&D. Under cross-licensing, as long as $\partial C / \partial x > -\frac{1}{2}$,
> firms invest less than the socially optimal amount in R&D and social welfare is also sub-optimal. Under mutual non-aggression, firms may invest too little, or, if the lead time advantage, f, is large enough, they may invest too much in R&D.

---

[15] A comparison of the Japanese and U.S. patent systems provides another suggestive example. The Japanese system grants many more (but narrower) patents per product; this is seen as encouraging cross-licensing (Ordover, 1991, Cohen et al, 2003).

R&D is socially sub-optimal under cross-licensing because the innovation rents are shared, making each firm's incentive too low. Under mutual non-aggression, a lead time advantage can provide stronger incentives and a more nearly socially optimal (or perhaps even supra-optimal) level of R&D. Note that this Proposition does not consider the full calculation of social welfare, which also needs to take deadweight loss into account; deadweight loss will, in general, be larger under cross-licensing than under non-aggression, where competition dissipates more rents to consumers.

Given that patenting costs are usually much smaller than R&D costs ($C<<x$), the condition, $\partial C/\partial x > -\frac{1}{2}$, will be met as long as C is not highly elastic with respect to x. This condition says, in effect, that innovation is the main driver of R&D, not the associated reduction in patenting cost, which is likely to be quite small. If it is small, then R&D incentives under cross-licensing will fall substantially short of the optimal level.

These results contrast with the simple patent race literature because the models in that literature assume that the winner of the innovation race is also the sole winner of a monopoly patent right. This is a very strong form of lead time advantage (although not market based). This strong incentive means that R&D may be excessive and, so, these models propose using patent policy instruments, such as patent term and scope, to reduce the incentive.[16] For example, a finite patent term limits the period over which monopoly rents can be collected.

But with patent thickets, even an infinite term may be insufficient to provide adequate incentives. In a sense, patent thickets may be viewed as an instance where the private actions of firms counteract policy. Policies based on the patent race models may be subject to a Lucas-type critique: if the government establishes a short patent term, firms will build portfolios of patents staggered over time to achieve a long term patent position. If the government establishes a narrow patent scope, firms will build large portfolios of many related patents to achieve a very broad patent position. These actions increase the market power of individual firms, taking the actions of other firms as a given. But when multiple firms pursue patent thicket strategies, the net result may be an equilibrium with *less* R&D. In effect, patents substitute for R&D as a source of competitive advantage.

---

[16] This literature includes Nordhaus (1969), Lee and Wilde (1980), Loury (1979) and Dasgupta and Stiglitz (1980), using patent term as an instrument and Gilbert and Shapiro (1990), Klemperer (1990), Gallini (1992), and Denicolo (1996) use patent scope or breadth.

Socially Optimal Standards

But this model suggests an alternative policy instrument, namely, patenting standards, S. Since patenting standards affect the choice between aggressive cross-licensing and non-aggression, they can be used to influence that choice. This may not achieve socially optimal R&D, but it may be second best optimal.

First, it is necessary to decide where each strategy generates greater R&D. Straightforward calculation shows that for a given set of parameters, there is a unique value of the lead time advantage f such that $I^N = I^L$. Let this value be $\tilde{f}$:

(7) $\tilde{f}(V^C/V^L) = f$ such that $I^N = I^L$.

Given (1), this variable is independent of S. Then for industries where $f > \tilde{f}$, non-aggression generates larger R&D incentives, since I^N increases with f; when $f < \tilde{f}$, cross-licensing generates higher levels of R&D. This is shown as a dashed line in Figure 3. Below this curve, cross-licensing generates higher R&D; above it, mutual non-aggression generates more R&D.

Assuming that mutual non-aggression is still socially preferable when it generates supra-optimal R&D, the socially optimal policy will select whichever equilibrium generates the higher level of R&D spending as long as the monopoly deadweight loss is not large. In general, deadweight loss will be greater under cross-licensing than under mutual non-aggression where firms compete. So in some cases, non-aggression may be a socially preferred outcome even when it generates lower R&D incentives. To keep things simple, I ignore this case. Then the socially optimal policy is:

> Proposition 4. Optimal patenting standards. Assuming that mutual non-aggression is socially preferable to cross-licensing in the case where $\hat{x}^N > \hat{x}^{Soc}$, and ignoring deadweight loss, then the socially optimal patenting standard for a single industry is determined as follows:

> If $f < \tilde{f}$, then the socially optimal standard is $S = 0$ (or S is sufficiently small), so that $C = 0$. Then all firms will cross-license.

> If $f \geq \tilde{f}$, then the socially optimal standard is sufficiently high so that $W^N \geq W^L$. Then no firms will assert patents.

An example of this policy can be seen in Figure 3. For an industry at point A, patenting standard S_1 is second best optimal, but S_0 is not.

In general, this policy suggests that patenting standards should be either very low or very high, depending on the industry. The first alternative, when $f < \tilde{f}$, corresponds roughly to a system of patent registration[17] at low cost, and strong, pro-patentholder enforcement. Such low patenting standards are desirable if lead time advantages are small and/or the industry is not very innovative ($h(\hat{x}^N) \ll r$) and competition dissipates most rents ($V^C \approx 0$).

If, instead, the industry is innovative (high h) or there are substantial lead time advantages or competition is "softer," then high patenting standards are preferable. This second policy sets patent standards high so that patenting is costly and few (or no) patents are acquired. Such high standards include the case where there are no patents for an industry, e.g., software subject matter exclusion (recall, I have ignored the problem of knock-off imitation).

This is, of course, a policy tailored to an individual industry. Policy analysts sometimes argue that industry-by-industry regulation is subject to capture by rent-seekers and therefore they prefer a "one-size-fits-all" approach to patent regulation. As I argue below, the one-size approach has its own political economic limitations. Nevertheless, if a single patenting standard has to apply to all industries, then a more complicated analysis must be performed. The empirical literature finds that many industries do have substantial lead time advantages (Levin et al, 1987, Cohen et al, 2000), suggesting a policy of high patenting standards. But a "one-size-fits-all" policy would have to temper high standards a bit so that socially wasteful patenting and "inventing around" expenditures are not too large in industries that do cross-license portfolios. For example, if the industries in Figure 3 are distributed uniformly in both dimensions, then S_1 might be an optimal single standard—most industries cross-license where this is preferable and vice versa, but some industries are mismatched.

Finally, note that this analysis suggests skepticism about the welfare effects of cross-licensing. Extensive cross-licensing may well be an indicator that patenting standards are too low and, consequently, R&D incentives are sub-optimal. In the context of a strictly static efficiency analysis, cross-licenses can permit more efficient outcomes (Shapiro, 2001). But excessive cross-licensing generated by low patenting standards is dynamically inefficient.

---

[17] Under a registration system, acquiring a patent simply involves registering it with the patent office; it is not examined. Britain and the U.S. initially had registration systems, but litigation rates were high, making expected costs high as well.

III. Extension to Sequential Innovation in a Single Market

Drastic Innovation with an Incumbent Firm

Next, I extend the basic model to the situation where one firm is an incumbent. This extension corresponds to the models in the literature on sequential replacement that study the persistence of monopoly (see Reinganum, 1989, for a review). The primary focus here, however, is to explore the distributional aspects of cross-licensing and patenting standards.

Suppose that everything is the same as in the duopoly case above except that firm A has an existing product that will be replaced by the new innovation that either firm A or B can invent ("drastic" innovation in the same market). Let firm A earn a stream of monopoly rents of $r \cdot u \cdot V^M$ on this existing product until the new innovation is marketed. The variable u is the ratio of the monopoly rent on the old technology to the rent on the new technology. It may be treated roughly as a measure of industry maturity. In more mature industries, innovations tend to be more incremental, resulting in smaller gains relative to the previous technology. Let $0 < u < 1$ so that the monopoly profits on the new innovation, V^M, are greater than the monopoly profits on the old innovation.

Since I wish to explore the distributional effects related to cross-licensing, I make several simplifying assumptions. First, I assume that each firm invests a fixed amount in R&D, $\bar{x}$, and innovates first (second) with a probability of $\frac{1}{2}$. The first innovation occurs instantly, so initial discounting can be ignored (but there is still a lead time advantage). This lets me focus on the effect of u on post-innovation outcomes (rents on the existing product may also diminish A's R&D incentives, but I ignore this effect here). Second, I let $C(S, n) = S \cdot n$; in other words, S corresponds to a fixed marginal cost of patenting. Finally, I assume that the patents acquired on the existing product do not intersect at all with the patents on the new product. On the one hand, this means that the patent position on the existing product cannot be challenged. More important, it also means that the incumbent firm's initial patents cannot be used to holdup the entrant if the entrant innovates. This is a strong simplifying assumption. In practice, most sequential improvements do infringe previous technology and because of this, the incumbent can obtain a larger share of the rents from the innovation with licensing. Below I show that incumbents obtain a majority of the rents *without* holdup, so allowing holdup in this case would only strengthen these distributional effects.

Industry maturity, u, affects the bargaining for a cross-license in the case that both firms win at litigation (mutual blocking). This happens because firm A continues to earn a profit stream of $r u V^M$ during negotiations, making A the more patient bargainer (or, alternatively, u alters the "threat point" of negotiations). Above, where the profits under a negotiated settlement to mutual blocking were $\left\{\frac{1}{2}V^M, \frac{1}{2}V^M\right\}$ in the symmetric model, now they are $\left\{\frac{1}{2}(1+u)V^M, \frac{1}{2}(1-u)V^M\right\}$ (see Appendix). This change in the litigation outcome tilts the cross-license in favor of firm A.

Given these additional assumptions, the equilibrium net expected profit for each firm under mutual non-aggression licensing is (assuming an interior solution)

(8) $W^N = \frac{1}{2} f V^M + (1-f)V^C - \bar{x}$.

This is equivalent to the solution in the symmetric model given in (3) with a zero discount rate during the Poisson race. An incumbent's profits on its existing product have no effect after innovation occurs.

With cross-licensing, the firms earn different profits after innovation, regardless of which firm makes the innovation. The equilibrium net expected profits are (see Appendix)

(9)
$$W_A^L = \frac{1}{2}\left(1 + q(\hat{n}_A) - q(\hat{n}_B) + q(\hat{n}_A)q(\hat{n}_B)u\right)V^M - S\cdot\hat{n}_A - \bar{x}$$
$$W_B^L = \frac{1}{2}\left(1 + q(\hat{n}_B) - q(\hat{n}_A) - q(\hat{n}_A)q(\hat{n}_B)u\right)V^M - S\cdot\hat{n}_B - \bar{x}$$

The values of $\hat{n}_A$ and $\hat{n}_B$ are found by solving first order conditions. In general, for $u > 0$, $\hat{n}_A > \hat{n}_B$ and $W_A^L > W_B^L$. The intuition behind this result is simple. Because the incumbent firm makes out relatively better when mutual blocking arises, firm A's incentive to obtain patents is greater. It obtains more patents and a correspondingly larger share of the profits of the innovation, regardless of whether the innovation is made by firm A or B.

Given this inequality, the boundary between non-aggression and licensing equilibria depends solely on the choice of firm A. Firm A will choose to assert patents in some situations where firm B would prefer a non-aggressive equilibrium. Now the boundary between equilibria is defined by

(10) $\bar{f}(S, V^C/V^L) = f$ such that $W^N = W_A^L$.

Cross-licensing occurs when it is in firm A's interest to cross-license.

As above, this paper does not consider deterrence strategies, although with positive u, firm A will generally choose deterrence over a greater range of parameter values.

Who cross-licenses?

These results indicate a distributional aspect to cross-licensing and to patent policy. Some empirical evidence suggests that industry maturity is related to aggressive cross-licensing and heavy patenting. In a study of 46 new technologies, Gort and Klepper (1982) find that patenting rates are highest in the most mature industries, although innovation rates are lower then. This could be because mature industries are more likely to engage in aggressive portfolio licensing as opposed to mutual non-aggression.

Interpreting u as a measure of industry maturity, the following result provides a possible theoretical explanation to Gort and Klepper's result (see Appendix).

Proposition 5. Industry Maturity and Cross licensing. In an industry with a larger value of u (more mature), the incumbent will obtain larger profits, that is, $\dfrac{d W_A^L}{d u} > 0$. Also, cross-licensing will occur over a larger range, that is, $\dfrac{d \bar{f}}{d u} > 0$.

Thus, in a general sense, incumbent firms make out better in mature industries and some incumbents in mature industries will choose cross-licensing because they find this more profitable. Low patenting standards can also make cross-licensing more profitable by reducing patenting costs, as in Proposition 2.

But do low patenting standards actually benefit incumbent firms? They do as long as the demand for patents is not too elastic (see Appendix):

Proposition 6. Incumbency and patenting standards. If

$$\frac{q'(n)}{n\,q''(n)} > -1 \quad \text{then} \quad \frac{d W_A^L}{d S} < 0.$$

Lower (higher) patenting standards increase (decrease) the profits of incumbents.

The ratio $\dfrac{q'(n)}{n\,q''(n)}$ is the elasticity of demand for patents in the absence of strategic interaction. The condition that this ratio be larger than -1 ensures that increases in the marginal cost of patenting will not decrease the total cost of patents. This is a reasonable assumption and it holds for many common distribution functions that might be used to model $q(\cdot)$.[18]

---

[18] The function q can be interpreted as a distribution function of the value of patents. Empirical studies based on patent renewal data typically find a highly skewed distribution of patent values. These studies use lognormal or Pareto distributions for the value of patents. Both of these functions meet the elasticity requirement in the appropriate range (the upper tail of the lognormal).

Incumbent firms do, indeed, have some incentive to promote lower patenting standards. Of course, a complete political economic analysis must consider the incentives of other groups, such as the patent bar, that may also have strong incentives regarding patenting standards. But this result does imply that a "one-size-fits-all" patent policy may not be immune from significant political concerns.

IV. Conclusion

A patent is held to be a form of "intellectual property" because it grants exclusive ownership over a productive asset. According to two arguments, ownership provides the strongest incentives for efficient use of the asset, either because monopoly ownership generates the largest rents, hence the strongest *ex ante* incentives (the Nordhaus incentive argument), or because a sole owner can most efficiently coordinate *ex post* development and commercialization (the Kitch prospect argument).

The model presented here suggests that patents may be an inefficient or defective property right if technologies are complex and patent standards are low. This is because patents do not, in fact, convey exclusive ownership over the *relevant productive assets* when a single technology involves large numbers of patents. The patent race model and the prospect model of patents depend on the crucial assumption that a productive innovation uniquely corresponds to just a single patent.

But when technologies are complex and standards are low, ownership is shared and the rents earned on an innovation are shared as well. This means that innovation incentives are too low even with efficient contracting and ignoring entry deterrence. Moreover, patents do not merely fail to provide sufficiently strong incentives in this case; they may also destroy the market-based incentives of lead time advantages. In effect, with low standards and complex technologies, patents serve to subsidize the losers of innovation races (paid by the winners), especially if those losers are large patent holders in mature industries. Schumpeterian competition is blunted.

Schumpeterian competition and more optimal R&D levels can be sustained in industries with lead time advantages if patenting standards are sufficiently high (or if few patents are granted, as in the software industry historically). But competition is not enhanced by reducing patenting standards, even if such changes "strengthen" patents in the sense of making patents less costly to obtain or otherwise favoring patent holders.

Much of the literature discussing patent thickets has focused on potential problems of transaction costs, holdup and vertical monopoly. This paper demonstrates that patent thicket strategies can discourage innovation even without these problems. This means that although cross-licensing and patent pools may resolve some problems of transaction cost and vertical monopoly, these institutions do not correct all problems associated with patent thickets.

Finally, this paper ignores the effect of patent thickets on entry. Patents may protect entrants (e.g., Hall and Ziedonis, 2001), but thickets force entrants to develop a portfolio quickly, possibly providing a barrier to entry. This effect remains to be explored.

References

Bessen, James. 2004. "Holdup and Licensing of Cumulative Innovations with Private Information," *Economics Letters*, v. 82, no. 3, pp. 321-26.

Bessen, James and Robert M. Hunt. 2004. "An Empirical Look at Software Patents," working paper.

Binmore, Ken, Ariel Rubinstein, and Asher Wolinsky. 1986. "The Nash Bargaining Solution in Economic Modelling," *RAND Journal of Economics*, v. 17, pp. 176-88.

Cohen, Wesley M., Richard R. Nelson, and John P. Walsh. 2000. "Protecting Their Intellectual Assets: Appropriability Conditions and why U.S. Manufacturing Firms Patent (or not)," National Bureau of Economic Research Working Paper 7552.

Cohen, Wesley M., Akira Goto, Akiya Nagata, Richard R. Nelson, John P. Walsh, (2002) "R&D spillovers, patents and the incentives to innovate in Japan and the United States", *Research Policy*, v. 31, pp. 1349-67.

Dasgupta, P. and Stiglitz, J. 1980. "Uncertainty, Industrial Structure, and the Speed of R&D," *Bell Journal of Economics*, 11, p. 1.

Denicolo, Vincenzo. 1996. "Patent Races and Optimal Patent Breadth and Length," *The Journal of Industrial Economics*, v. 44 pp. 249-265.

Draper, George O. 1903. Textile Texts, 2nd ed. (Hopedale, Mass.) quoted in William Mass. 1989, "Mechanical and Organizational Innovation: The Drapers and the Automatic Loom," *Business History Review*, v. 63, p. 876-929.

Eswaran, Mukesh. 1993. "Cross-licensing of competing patents as a facilitating device," *Canadian Journal of Economics*, v. 27 pp. 689-708.

Fershtman, Chaim and Morton I. Kamien. 1992. "Cross licensing of complementary technologies," *International Journal of Industrial Organization*, v. 10 pp. 329-348.

Gallini, Nancy T. 1992. "Patent Policy and Costly Imitation," *RAND Journal of Economics*, v. 23, pp. 52-63.

Gallini, Nancy T. 2002. "The Economics of Patents: Lessons from Recent U.S. Patent Reform," *Journal of Economic Perspectives*, v. 16, n. 2, pp. 131-54.

Gilbert, Richard and David M. G. Newbery. 1982. "Preemptive Patenting and the Persistence of Monopoly," *American Economic Review*, v. 72, pp. 514-526.

Gilbert, Richard and Carl Shapiro. 1990. "Optimal Patent Length and Breadth," *RAND Journal of Economics*, v. 21, pp. 106-112.

Gort, M. and Klepper, S. 1982. "Time paths in the diffusion of product innovations," *The Economic Journal*, v. 92, p. 630.

Grindley, Peter C. and David J. Teece. 1997. "Managing Intellectual Capital: Licensing and Cross-Licensing in Semiconductors and Electronics," *California Management Review,* v. 29, pp. 8-41.

Hall, Bronwyn H. and Rosemary Ham Ziedonis. 2001. "The Patent Paradox Revisited: An Empirical Study of Patenting in the U.S. Semiconductor Industry, 1979-1995" *RAND Journal of Economics.* 32:1, pp. 101-128.

Heller, Michael A. and R.S. Eisenberg. 1998. "Can Patents Deter Innovation? The Anticommons in Biomedical Research" *Science*. 280, pp. 698-701.

Kitch, Edmund W. 1977. "The Nature and Function of the Patent System," *Journal of Law and Economics*, v. 20, pp. 265-290.

Klemperer, Paul. 1990. "How Broad Should the Scope of Patent Protection Be?" *RAND Journal of Economics*, v. 21, pp. 113-130.

Lee, T. and Wilde, L. 1980. "Market Structure and Innovation: A Reformulation," *Quarterly Journal of Economics*, 94, p. 429.

Lerner, Josh and Jean Tirole. 2002. "Efficient Patent Pools," working paper.

Lerner, Josh, Marcin Strojwas, and Jean Tirole. 2003. "The Structure and Performance of Patent Pools: Empirical Evidence," working paper.

Levin, Richard C. 1982. "The Semiconductor Industry," in Richard R. Nelson, ed., *Government and Technical Progress: A Cross-Industry Analysis* (Pergamon Press, 1982).

Levin, Richard C., Alvin K. Klevorick, Richard R. Nelson, and Sidney G. Winter. 1987. "Appropriating the Returns from Industrial Research and Development," *Brooking Papers on Economic Activity, 3,* pp. 783-820.

Loury, G. 1979. "Market Structure and Innovation," *Quarterly Journal of Economics*, 93, p. 395.

Magrab, E. Brendan, 1993. "Patent Validity Determinations of the ITC: Should U.S. District Grant Them Preclusive Effect?," 75 *Journal of the Patent & Trademark Office Society* 125, 127-35.

Mass, William. 1989. "Mechanical and Organizational Innovation: The Drapers and the Automatic Loom," *Business History Review*, v. 63, p. 876-929.

Merges, Robert P. 1999. "Institutions for Intellectual Property Transactions: The Case of Patent Pools," Boalt Hall School of Law Working Paper, University of California, Berkeley.

Norhaus, William D. 1969. "An Economic Theory of Technological Change," *American Economic Review*, v. 59, no. 2, pp, 18-28.

Ordover, Janusz. 1991. "A Patent System for Both Diffusion and Exclusion," *Journal of Economic Perspectives*, v. 5, pp. 43-60.

Post, Robert C. 1976. " 'Liberalizers' versus 'Scientific Men' in the Antebellum Patent Office," *Technology and Culture*, v. 17, pp. 24- 54.

Reinganum, Jennifer F. 1989. "The Timing of Innovation: Research, Development and Diffusion," in Richard Schmalensee and Robert D. Willig, eds., Handbook of Industrial Organization, Amsterdam: North-Holland.

Shapiro, Carl. 2001. "Navigating the Patent Thicket: Cross Licenses, Patent Pools, and Standard-Setting," in Adam Jaffe, Joshua Lerner, and Scott Stern, eds., *Innovation Policy and the Economy*, National Bureau of Economic Research.

Smith, Roger. 1990. quoted in Boyer, Chuck, "The Power of the Patent Portfolio," *Think* (IBM), No. 5, pp. 10-11.

Tilton, J.E. 1971. *International Diffusion of Technology: The Case of Semiconductors*, Washington, D.C.: Brookings Institution.

Tirole, Jean. 1999. "Incomplete Contracts: Where Do We Stand?" *Econometrica,* v. 67, n. 4, pp. 741-781.

U.S. Patent and Trademark Office. 1994. "Public Hearings on Patent Protection for Software-Related Inventions," available at http://www.eff.org/IP/ca_softpatent.tesimony.

Appendix

Equilibria and First Order Conditions

Value of cross-license

I derive the bargaining solution for the cross-license and first order conditions for the asymmetric case. The symmetric case is the instance where industry maturity $u = 0$. First, I solve the bargaining problem for the situation where firms have mutually blocking patents. In this case,

the "threat point" to a Nash bargaining solution for firms A and B is $\{u \cdot V^M, 0\}$; by cooperating,

the firms can realize joint profits of V^M. The bargaining solution is then

$\{\frac{1}{2}(1+u)V^M, \frac{1}{2}(1-u)V^M\}$. The same solution can be obtained with a Rubinstein-Stahl model

where firm A earns rents of $r \cdot u \cdot V^M \, dt$ each bargaining cycle of infinitesimal duration, dt

(Binmore et al, 1986).

      Given this solution to the case of mutual blocking, I calculate the total expected value of

litigation for each firm, T_i, $i = A, B$. Given legal costs, L, and probability of winning at

litigation, $q(n_i)$, this threat point is

(A1)

$$\{T_A, T_B\} = \begin{cases} (1-q(n_A))(1-q(n_B))V^C + q(n_A)(1-q(n_B))V^M + \frac{1}{2}q(n_A)q(n_B)(1+u)V^M - L, \\ (1-q(n_A))(1-q(n_B))V^C + q(n_B)(1-q(n_A))V^M + \frac{1}{2}q(n_A)q(n_B)(1-u)V^M - L \end{cases}$$

On the other hand, if the firms cooperate, their joint profits are V^M. Solving this Nash

bargaining problem, the cross-license provides profits, ignoring patenting and R&D costs, of

(A2) $$\{\Pi_A, \Pi_B\} = \begin{cases} \frac{1}{2}(1+q(n_A)-q(n_B)+q(n_A)q(n_B)u)V^M, \\ \frac{1}{2}(1+q(n_B)-q(n_A)-q(n_A)q(n_B)u)V^M \end{cases}$$

Asymmetric licensing equilibrium

      This value can be used to calculate the expected present value under the cross-licensing

equilibrium. The calculation here is somewhat different under the symmetric and asymmetric

models in the text because the asymmetric model does not involve time discounting. For the

asymmetric case, the value of cross licensing is by straightforward calculation

(A3)
$$W_A^L = \frac{1}{2}(1+q(\hat{n}_A)-q(\hat{n}_B)+q(\hat{n}_A)q(\hat{n}_B)u)V^M - S \cdot \hat{n}_A - \bar{x}$$
$$W_B^L = \frac{1}{2}(1+q(\hat{n}_B)-q(\hat{n}_A)-q(\hat{n}_A)q(\hat{n}_B)u)V^M - S \cdot \hat{n}_B - \bar{x}$$

where $\hat{n}_A$ and $\hat{n}_B$ solve the first order conditions:

(A4)   (a) $\frac{1}{2}(1+q(\hat{n}_B)u)V^M q'(\hat{n}_A) = S$ (b) $\frac{1}{2}(1-q(\hat{n}_A)u)V^M q'(\hat{n}_B) = S$.

Comparing these two conditions, for positive u, $\hat{n}_A$ and $\hat{n}_B$, it must be true that

$q'(\hat{n}_B) > q'(\hat{n}_A)$. Given the concavity of q, this means that $\hat{n}_A > \hat{n}_B$. And from this it can easily

be shown that $W_A^L > W_B^L$.

Symmetric licensing equilibrium

For the symmetric case, time discounting matters and $u = 0$. Assuming that the firms cross-license as soon as one of them discovers the innovation, the expected value for the *i*th firm, $i = A, B$, taking the opponent's allocation as given, is

$$W_i^L = \max_{x_i, n_i}\left[\int_0^\infty e^{-(r+h(x_A)+h(x_B))}\left(h(x_A)+h(x_B)\right)\Pi_i \, dt \; - \; x_i \; - \; C(S, n_i, x_i)\right]$$

(A5)

$$= \max_{x_i, n_i}\left[\frac{\left(h(x_A)+h(x_B)\right)\Pi_i}{r+h(x_A)+h(x_B)} \; - \; x_i \; - \; C(S, n_i, x_i)\right]$$

By the symmetry of the problem, the Nash equilibrium will have $\hat{n}_A = \hat{n}_B = \hat{n}$ and $\hat{x}_A = \hat{x}_B = \hat{x}^L$. Taking first derivatives and applying the symmetry condition, the Nash equilibrium has an expected value for each firm of W^L with first order conditions:

$$W^L \; = \; \frac{V^L}{1 + r\big/2h(\hat{x}^L)} \; - \; \hat{x}^L \; - \; C(S, \hat{n}, \hat{x})$$

(A6) $\quad \hat{x}^L: \quad I^L h'(\hat{x}^L) = \left(r + 2h(\hat{x}^L)\right)^2, \quad I^L \equiv \dfrac{rV^L}{1 + \partial C/\partial x}$

$$\hat{n}: \quad \frac{V^L q'(\hat{n})}{1 + r\big/2h(\hat{x}^L)} = \frac{\partial C(S, \hat{n}, \hat{x})}{\partial n}$$

and *I* is the R&D incentive.

Mutual Non-aggression equilibrium

For mutual non-aggression in the symmetric case,

$$W_i^N = \max_{x_i}\left[\int_0^\infty e^{-(r+h(x_A)+h(x_B))}\left(f V^M h(x_i) \; + \; (1-f)\left(h(x_A)+h(x_B)\right)V^C\right)dt \; - \; x_i\right]$$

(A7)

$$= \max_{x_i}\left[\frac{f V^M h(x_i) \; + \; (1-f)\left(h(x_A)+h(x_B)\right)V^C}{r+h(x_A)+h(x_B)} \; - \; x_i\right]$$

Again, a symmetry condition applies, $\hat{x}_A = \hat{x}_B = \hat{x}^N$. Taking derivatives and applying the symmetry condition yields a Nash equilibrium expected profit and first order condition

$$(A8) \qquad W^N = \frac{\frac{1}{2}fV^M + (1-f)V^C}{1 + r/2h(\hat{x}^N)} - \hat{x}^N \qquad .$$

$$\hat{x}^N : \quad I^N h'(\hat{x}^N) = \left(r + 2h(\hat{x}^N)\right)^2, \quad I^N \equiv (r + h(\hat{x}^N))fV^M + r(1-f)V^C$$

Social planner

Finally, consider the socially optimal level of R&D. The social planner is indifferent to which firm discovers the innovation. With two firms, the social planner will choose to optimize

$$(A9) \qquad W^{Soc} = \max_{x_A, x_B}\left[\int_0^\infty e^{-(r + h(x_A) + h(x_B))}\left(h(x_A) + h(x_B)\right)V^{Soc}\, dt - x_A - x_B\right]$$

$$= \max_{x_A, x_B}\left[\frac{\left(h(x_A) + h(x_B)\right)V^{Soc}}{r + h(x_A) + h(x_B)} - x_A - x_B\right]$$

Solving, again the solution is symmetric, $\hat{x}_A = \hat{x}_B = \hat{x}^{Soc}$

$$(A10) \qquad W^{Soc} = \frac{V^{Soc}}{1 + r/2h(\hat{x}^{Soc})} - 2\hat{x}^{Soc}$$

$$\hat{x}^{Soc} : \quad I^S h'(\hat{x}^{Soc}) = \left(r + 2h(\hat{x}^{Soc})\right)^2, \quad I^{Soc} \equiv rV^{Soc}$$

Outline of Proofs of Propositions

Proposition 2

$$\frac{d\bar{f}(S)}{dS} = -\frac{1 + r/2h(\hat{x}^N)}{\frac{1}{2}V^M - V^C} \cdot \frac{dC(S, \hat{n}(S), \hat{x}(S))}{dS}. \text{ If } V^C < \frac{1}{2}V^M \text{ and}$$

$\dfrac{dC(S, \hat{n}(S), \hat{x}(S))}{dS} > 0$, this will be negative. The changes in equilibria follow.

Proposition 5

Straightforward calculation of the implicit derivative reveals $\dfrac{dW_A^L}{du} > 0$. Given this result, and because W^N is independent of u, an increase in u means greater value of W_A^L such that $\bar{f}$ must be larger in order to maintain the equality condition in (10), so $\dfrac{d\bar{f}}{du} > 0$.

Proposition 6

First, it is helpful to define reaction functions

$$R_A(n_B, S) = \arg\max_{n_A}\left(\Pi_A(n_A, n_B) - S \cdot n_A\right)$$

(A11)

$$R_B(n_A, S) = \arg\max_{n_B}\left(\Pi_B(n_A, n_B) - S \cdot n_B\right)$$

Then the Nash equilibrium condition can be written

(A12) $\quad \hat{n}_B = R_B\left(R_A(\hat{n}_B, S), S\right), \qquad \hat{n}_A = R_A(\hat{n}_B, S)$

Taking the implicit derivative,

(A13) $\quad \dfrac{d\,\hat{n}_B}{d\,S} = \left(\dfrac{\partial R_B}{\partial S} + \dfrac{\partial R_B}{\partial n_A}\dfrac{\partial R_A}{\partial S}\right) \Big/ \left(1 - \dfrac{\partial R_B}{\partial n_A}\dfrac{\partial R_A}{\partial n_B}\right).$

Then, evaluating the partial derivates as implicit derivatives from (A4), straightforward calculation yields the following useful result:

Lemma. If $\dfrac{q'(n)}{n\,q''(n)} > -1$, then $-S\cdot\dfrac{d\,\hat{n}_B}{d\,S} < \hat{n}_B$.

Then,

$$\dfrac{d\,W_A^L}{d\,S} = \dfrac{\partial W_A^L}{\partial \hat{n}_A}\dfrac{d\,\hat{n}_A}{d\,S} + \dfrac{\partial W_A^L}{\partial \hat{n}_B}\dfrac{d\,\hat{n}_B}{d\,S} - \hat{n}_A$$

(A14)

$$= -S\cdot\dfrac{d\,\hat{n}_B}{d\,S} - \hat{n}_A$$

the latter by the envelope theorem and (A4b). Then by the Lemma and since $\hat{n}_A > \hat{n}_B$, it follows that

(A15) $\quad \dfrac{d\,W_A^L}{d\,S} < 0.$

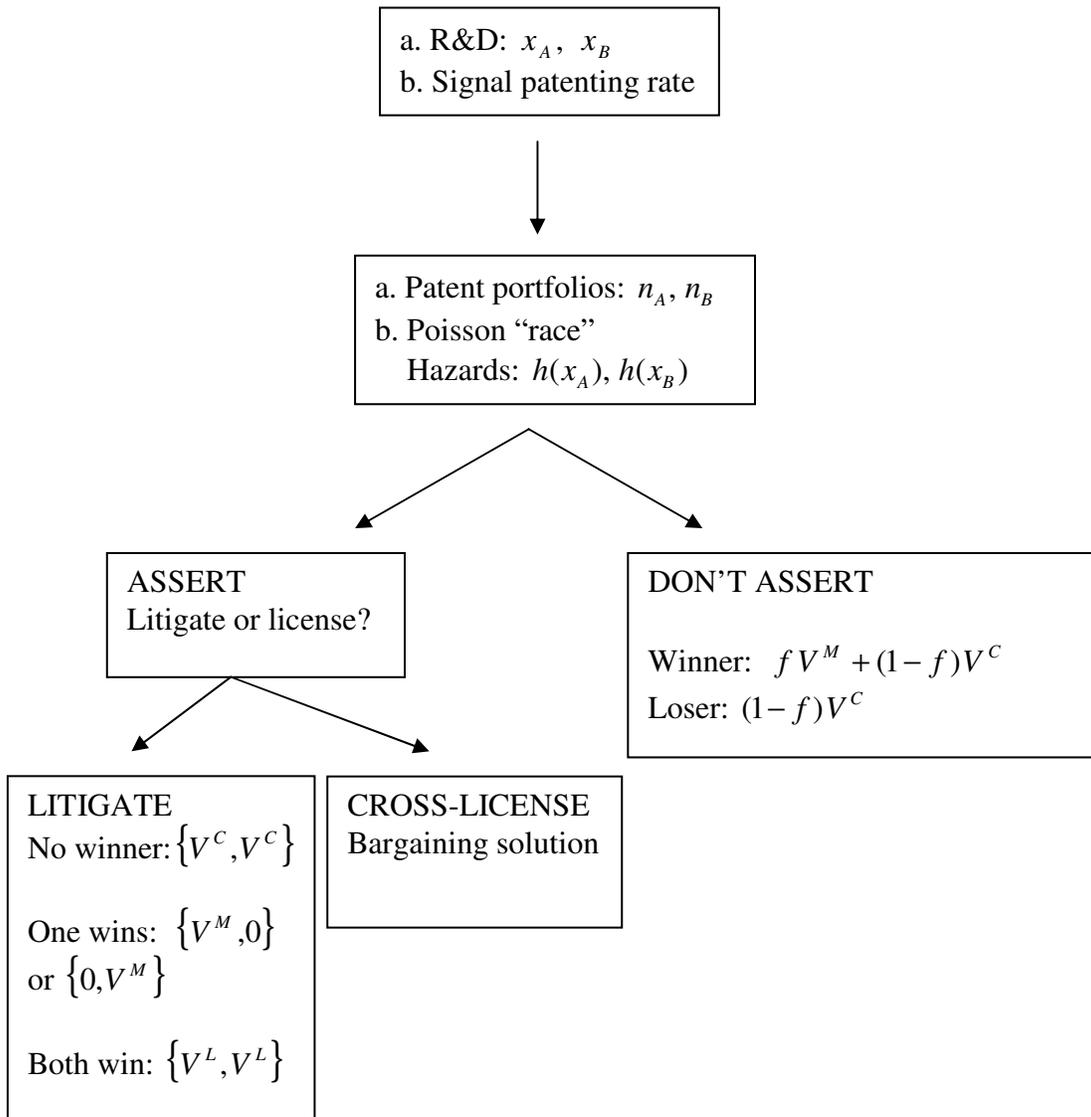Figure 1. Innovation Race Model

a. R&D: x_A, x_B
b. Signal patenting rate

a. Patent portfolios: n_A, n_B
b. Poisson "race"
   Hazards: $h(x_A)$, $h(x_B)$

ASSERT
Litigate or license?

DON'T ASSERT

Winner: $f V^M + (1 - f)V^C$
Loser: $(1 - f)V^C$

LITIGATE
No winner: $\{V^C, V^C\}$

One wins: $\{V^M, 0\}$
or $\{0, V^M\}$

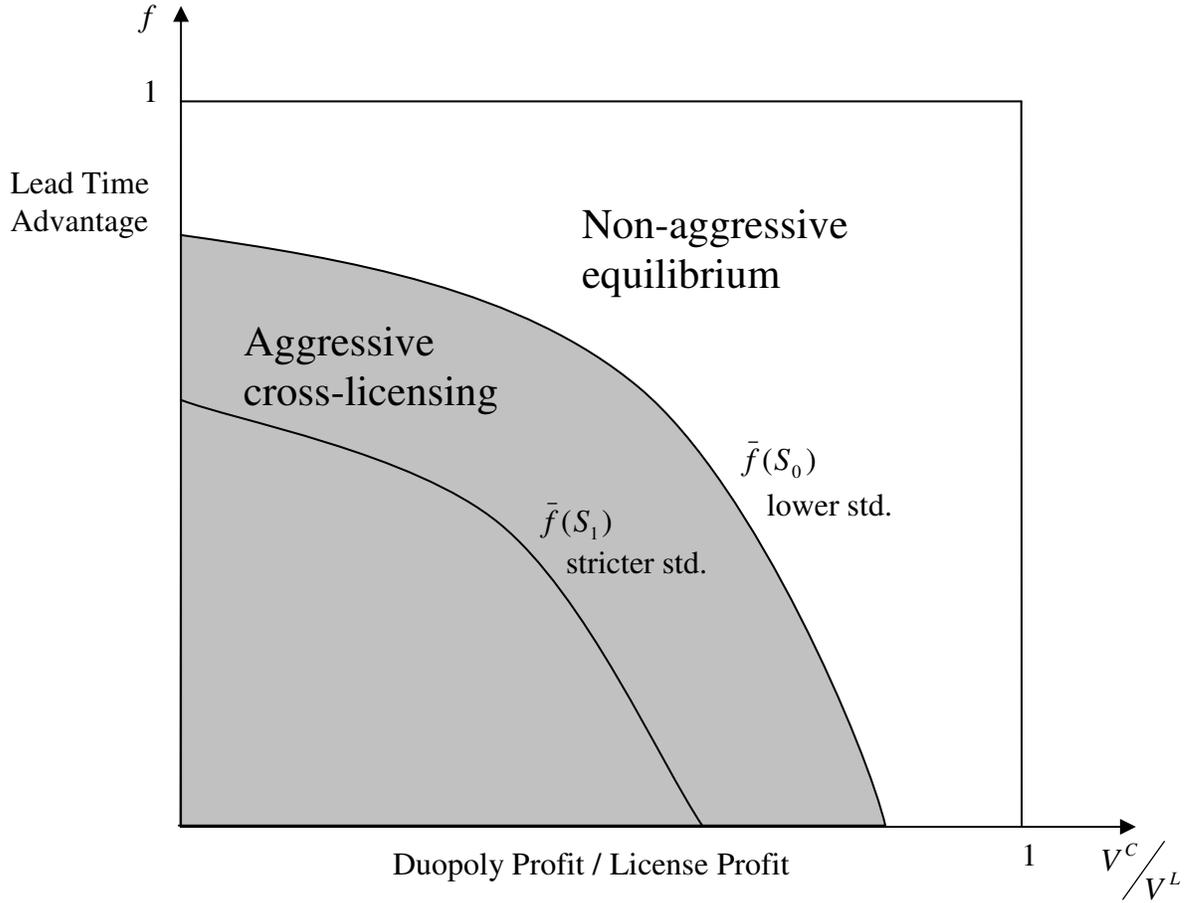Both win: $\{V^L, V^L\}$

CROSS-LICENSE
Bargaining solution

Figure 2. Phase Diagram of Cross-Licensing and Mutual Non-aggression

Notes: $\bar{f}(S, V^C/V^L) = f$ such that $W^N = W^L$

$S_1 > S_0$

The shaded area represents the region where the firms choose the cross-licensing equilibrium under patentability standard S_0; this is also the area where profits under cross-licensing are greater, $W^L > W^N$ under this standard. Firms choose a non-aggressive equilibrium in the unshaded area.

Figure 3. Phase Diagram Showing R&D Incentive Zones
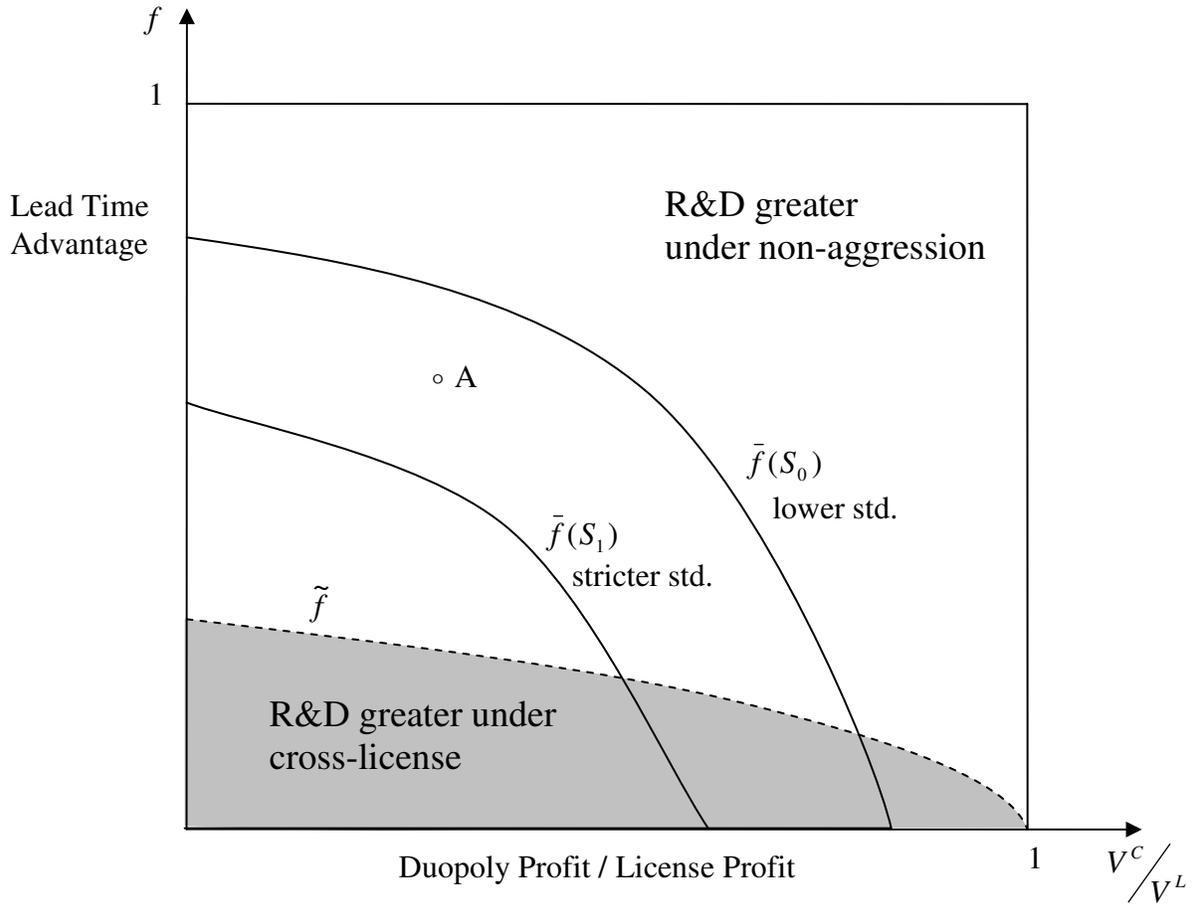
Notes: $\bar{f}(S, V^C/V^L) = f$ such that $W^N = W^L$

$\tilde{f}(V^C/V^L) = f$ such that $I^N = I^L$

$S_1 > S_0$

The shaded area represents the region where R&D incentives are greater under cross-licensing, $I^L > I^N$. In the unshaded area, R&D incentives are greater under a non-aggressive equilibrium.